2020 e-Book Edition

PROOFS BY INDUCTION

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This author of this document is Trishit Dedhia.

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If you have any doubts, queries or feedback, you may reach the CrackBox Team using email: crackboxofficial@gmail.com.

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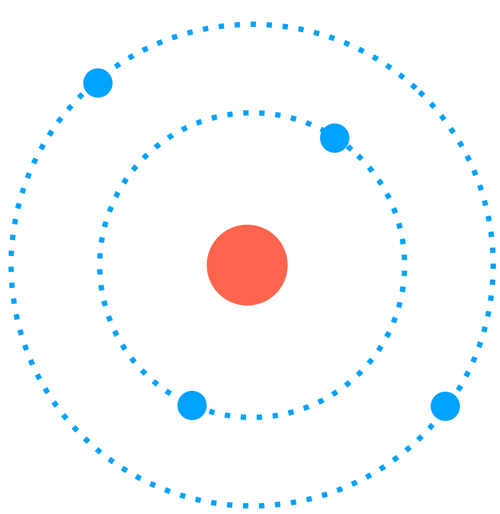

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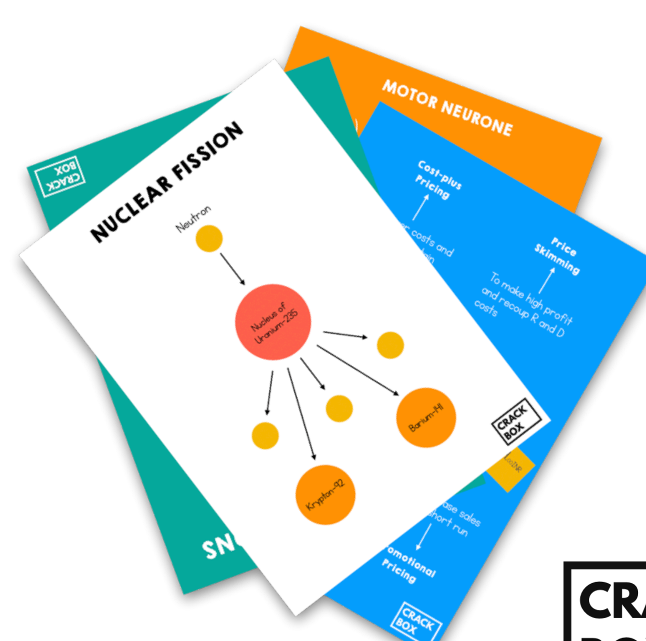

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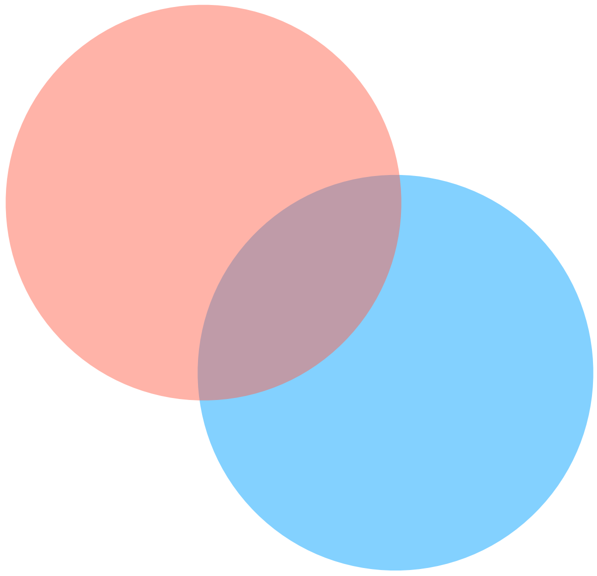

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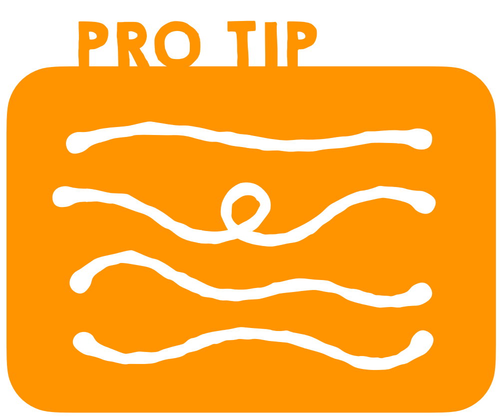

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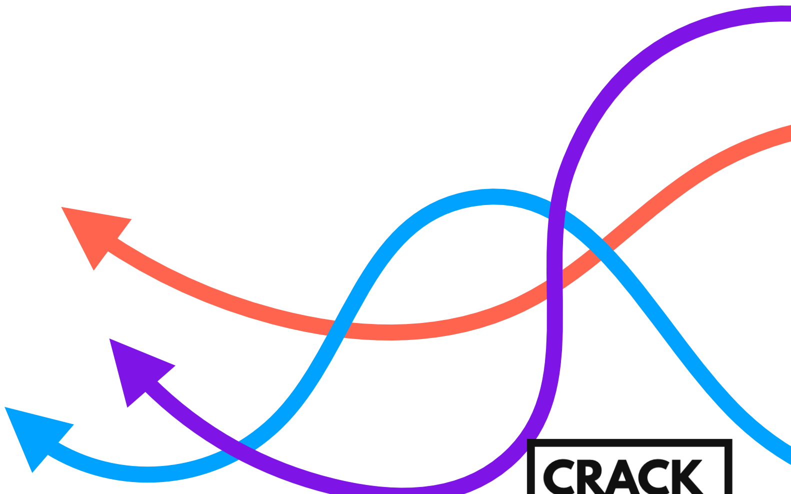

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We use mathematical induction to prove that two sides of an equation are equal to each other. In other words, LHS = RHS.

Let's take this problem for example:

$$1 + 2 + 3 + ... + n = \frac{n(n+1)}{2}$$

Where $1 + 2 + 3 + ... + n$ can be denoted as S_n.

We have divided the entire process into 3 steps:

STEP 1: Prove for base case.

STEP 2: Assume p(k) is true.

STEP 3: Prove p(k+1) is true.

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Step 1

We take the base case for n and check whether the equation stands true. Since S_n has a domain of all positive integers the base case will be $n = 1$.

p(1) is just a method of denoting the case for which n = 1.

p(1)

LHS	RHS
1	$\frac{1(1+1)}{2}$
1	$= 1$

Hence proved that p(1) is true as LHS = RHS.

PRO TIP

Solve p(2), and p(3). In rough, to understand how p(k) works.

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Step 2

Now we assume the equation is true for p(k). In this step we basically we substitute n with a variable k.

Assuming p(k) is true:

$$1 + 2 + \ldots + k = \frac{k(k + 1)}{2}$$

Step 3

Still assuming p(k) is true we proceed to p(k+l) where again we substitute n with k + l.

p(k+1):

$$1 + \ldots + k + k + 1 = \frac{(k + 1)((k + 1) + 1)}{2}$$

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From step 2, we recall p(k). And hence we can substitute that in the equation.

LHS:

$$1 + \ldots + k + k + 1 = \frac{k(k+1)}{2} + k + 1$$

If we simplify this expression further, we will be able to prove that LHS = RHS.

$$= \frac{k(k+1)}{2} + \frac{2(k+1)}{2}$$

$$= \frac{k(k+1) + 2(k+1)}{2}$$

$$= \frac{(k+1)(k+2)}{2}$$

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Taking closer attention, we can reframe this same expression to finally prove it.

$$\frac{(k + 1)(k + 2)}{2}$$

$$\frac{(k + 1)((k + 1) + 1)}{2}$$

This is the expression for when n = k + 1

Don't forget to conclude the proof.

Thus, p(k + 1) is true for when p(k) is true. Therefore, p(n) is true for all positive integers.

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Here is the entire sample answer for quick recap and revision.

p(1)

LHS	RHS
1	$\frac{1(1 + 1)}{2}$
1	$= 1$

Hence proved that p(1) is true as LHS = RHS.

Assuming p(k) is true:

$$1 + 2 + ... + k = \frac{k(k + 1)}{2}$$

p(k+1):

$$1 + ... + k + k + 1 = \frac{(k + 1)((k + 1) + 1)}{2}$$

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LHS:

$$1 + \ldots + k + k + 1 = \frac{k(k+1)}{2} + k + 1$$

$$= \frac{k(k+1)}{2} + \frac{2(k+1)}{2}$$

$$= \frac{k(k+1) + 2(k+1)}{2}$$

$$= \frac{(k+1)(k+2)}{2}$$

$$= \frac{(k+1)((k+1)+1)}{2}$$

Thus, p(k + 1) is true for when p(k) is true. Therefore, p(n) is true for all positive integers.

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